

For the groundstate wavefunction:

$$\Psi_0 = N_0 e^{-\frac{dx^2}{2}} \rightarrow E_0 = \frac{1}{2} \hbar \omega_0$$

$$\Psi_n = N_n H_n \left(\sqrt{\frac{1}{2}} x \right) e^{-\frac{dx^2}{2}} \rightarrow E_n = ?$$

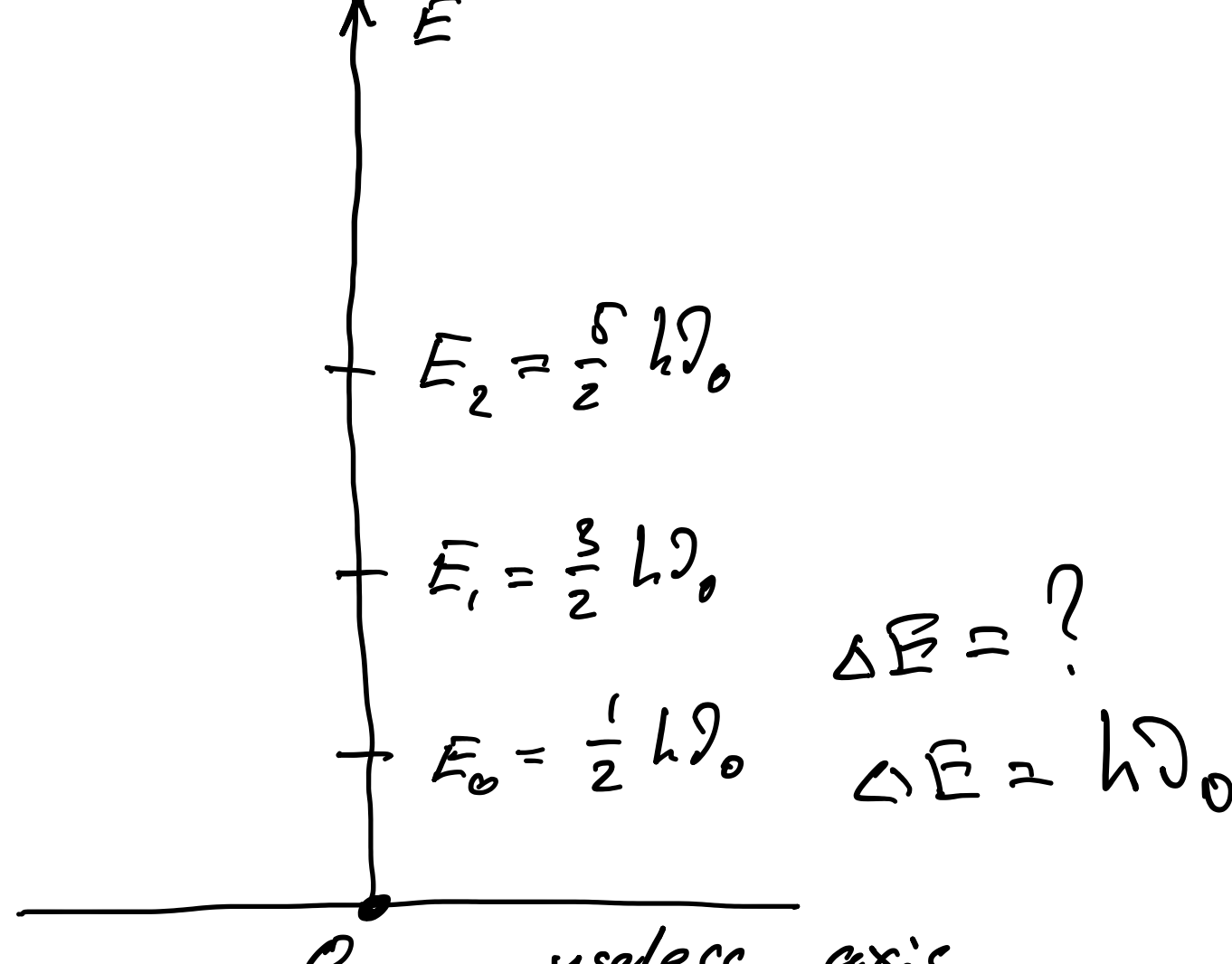
We will simply need to input it to Schrödinger equation:

$$\Psi_1 = N_1 x e^{-\frac{dx^2}{2}} \rightarrow E_1 = \frac{3}{2} \hbar \omega_0$$

$$\Psi_2 = N_2 (4dx^2 - 2) e^{-\frac{dx^2}{2}} \rightarrow E_2 = \frac{5}{2} \hbar \omega_0$$

$$\vdots$$

$$\Psi_n = N_n H_n \left(\sqrt{\frac{1}{2}} x \right) e^{-\frac{dx^2}{2}} \rightarrow E_n = \left(n + \frac{1}{2} \right) \hbar \omega_0$$

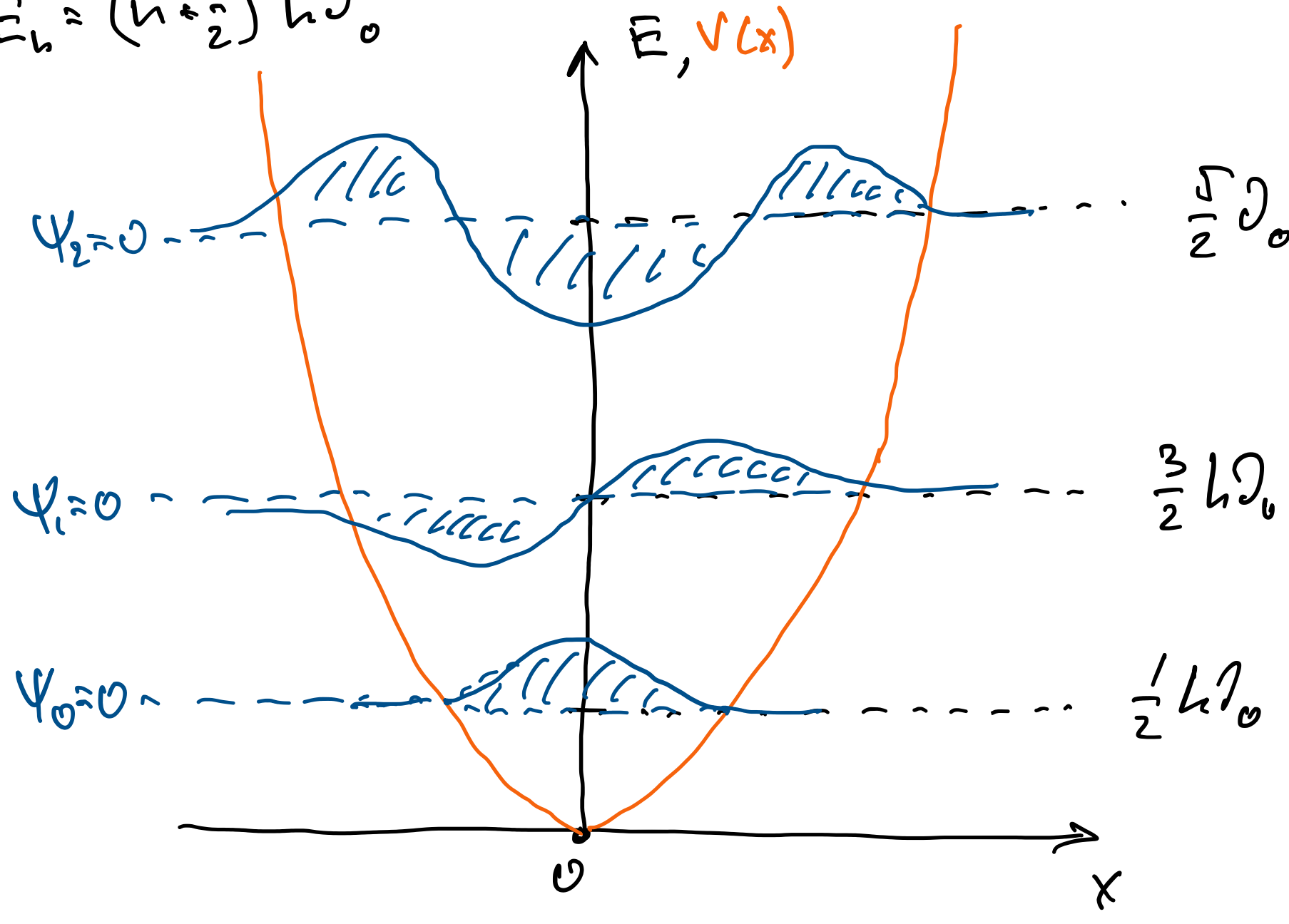


If we to summarize the picture for harmonic oscillator

$$V(x) = \frac{1}{2} k x^2$$

$$\Psi_n(x) = N_n H_n \left(\sqrt{\frac{1}{2}} x \right) e^{-\frac{dx^2}{2}}$$

$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega_0$$



Let's see how selection rules work for HO

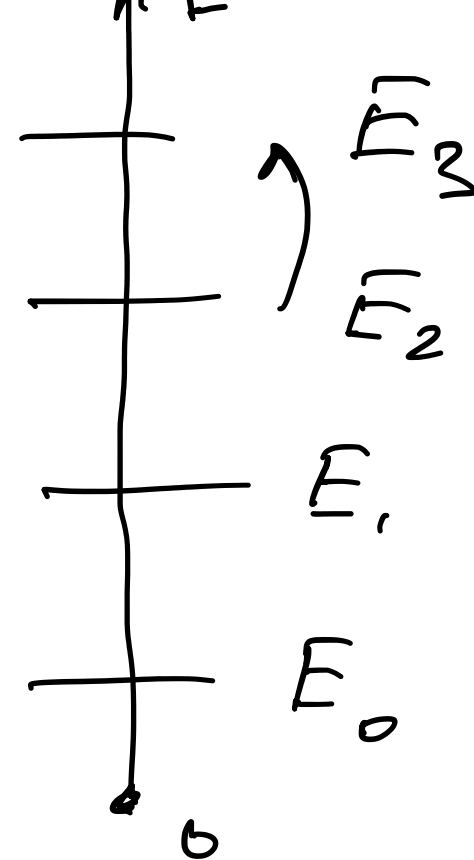
part of Fermi's Golden Rule:

$$n \rightarrow n' \int \Psi_n^* \mu_z \Psi_{n'} dx$$

The problem here is μ_z .

It is not a constant anymore,

$$\text{as } \mu = \mu_0 + \frac{\partial \mu}{\partial x} x$$



$$\int \Psi_n^* \mu_z \Psi_{n'} dx = \int \Psi_n^* \left(\mu_0 + \frac{\partial \mu}{\partial x} x \right) \Psi_{n'} dx =$$

$$= \mu_0 \int \Psi_n^* \Psi_{n'} dx + \frac{\partial \mu}{\partial x} \int \Psi_n^* x \Psi_{n'} dx$$

0

1) if $\frac{\partial \mu}{\partial x} = 0$, then transition is forbidden

obvious $\frac{\partial \mu}{\partial x} = 0 \rightarrow$ no oscillation

CO vs H₂

$$2) \int \Psi_n^* x \Psi_{n'} dx = 0$$

$$\Psi_n = N_n H_n \left(\sqrt{\frac{1}{2}} x \right) e^{-\frac{dx^2}{2}} \quad \text{who cares?}$$

$$\int N_n^* H_n \times N_{n'} H_{n'} e^{-dx^2} = N_n^* N_{n'} \int H_n \times H_{n'} e^{-dx^2} =$$

$$H_{n+1} = 2x H_n - 2n H_{n-1}$$

$$x H_n = \frac{1}{2} H_{n+1} - n H_{n-1}$$

$$= N_n^* N_{n'} \int H_n \left(\frac{1}{2} H_{n'+1} - n' H_{n'-1} \right) e^{-dx^2} dx =$$

$$= A \int H_n H_{n'+1} dx - B \int H_n H_{n'-1} dx$$

$\neq 0$

$\neq 0$

$n = n' + 1$

$n = n' - 1$

$$\boxed{\Delta n = \pm 1} \quad \nabla$$

0